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Analysis of faulting in porous sandstones

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Abstract—Faults in porous sandstones occur in three forms: *deformation bands* about 1-mm thick and tens of m long and across which offsets are a few mm; *zones of deformation bands* constituted of many closely spaced deformation bands across which offsets are a few cm or dm; and *slip surfaces*, that is, distinct surfaces within zones of deformation bands across which offsets are a few m to a few tens of m. Deformation bands represent highly localized deformation; analogous localization within a field of homogeneous deformation is theoretically possible in inelastic materials with certain ranges of constitutive parameters. Crushing and consolidation of sandstone within a band cause the material there to become stiffer than the surrounding porous sandstone. A zone of deformation bands behaves mesoscopically much as a stiff inclusion in a soft matrix. According to the constitutive model assumed to investigate the formation of deformation bands, an instability can develop, and strain increments within the zone of deformation bands can become boundlessly large when the far-field stresses reach critical values. This instability is here associated with the formation of slip surfaces.

INTRODUCTION

UNTIL recently, faults have been analyzed as though they were planar surfaces of discontinuity of displacement (see for example, Odé 1960, Varnes 1962). Yet deformation bands and zones of deformation bands, two types of faults in the Entrada and Navajo sandstones (Aydin 1978, Aydin & Johnson 1978), are finite in thickness and the strain is distributed through the thickness. Several investigators have recently developed theoretical models to explain the formation of faults and shear bands of finite thickness. Brady (1974) attempted to analyze the growth of faults in a brittle–elastic material containing cracks, representing a fault zone as a soft inclusion. Argon (1975) analyzed the development of deformation bands in strain-softening polymers. Rice, Rudnicki and Cleary showed that the inhomogeneous nature of deformation banding can be explained as a bifurcation in the macroscopic constitutive relations of homogeneous deformation for brittle or plastic materials (Rice 1975, Rudnicki & Rice 1975, Cleary & Rudnicki 1976, Cleary 1976, Rudnicki 1977, Rice 1979).

The assumptions in the analyses of Rice, Rudnicki and Cleary appear to be consistent with some of our observations of faulting in the Navajo and Entrada sandstones and with the common experimental observation that the stress–strain curves for sedimentary rocks under confining pressures greater than about 0.5 kb tend to be concave downward (e.g. Handin & Hager 1957). There is, however, no relevant experimental information about many of the constitutive parameters that enter the theory. It is clear that relevant experiments are essential. Yet we can learn something about deformation bands in the Entrada and Navajo sandstones by following the approach of Rudnicki & Rice (1975), and something about the formation of slip surfaces by following the approach of Rudnicki (1977) and Rice (1979).

In previous papers we have described three types of faults in porous sandstones (Aydin 1978, Aydin & Johnson 1978) and have documented their patterns (Aydin & Reches 1982). In this paper we collect the observations we believe to be the most important and compare them to results of analyses of idealized processes of faulting. Our observations of faulting in sandstones can suggest only rough estimates of the ranges of some of the parameters.

NATURE OF FAULTING IN POROUS SANDSTONE

Faults in the Navajo and Entrada sandstones in SE Utah have been described in detail in other papers (Aydin 1978, Aydin & Johnson 1978, Aydin & Reches 1982) so that the descriptions here will be brief. Both the Navajo and Entrada sandstones are friable, being composed primarily of quartz and feldspar, and quite porous, pores constituting about 25% of the rock volume. Three forms of faults are recognized in these sandstones: deformation bands, zones of deformation bands and slip surfaces. Each form occurs in dip–slip, strike–slip and oblique– slip. Small faults occur as deformation bands, about 1-mm thick, in which pores collapse and sand grains fracture and along which there are shear displacements of the order of a few mm or cm. Two or more adjacent deformation bands that share the same strike and dip form a zone of deformation bands. A zone becomes thicker by addition of new bands, side by side.

Finally, slip surfaces, through-going surfaces of discontinuity in displacement, form at either edge of zones of highly concentrated deformation bands. In the San Rafael Desert (SE Utah, U.S.A.) slip surfaces accommodate large displacements in the order of several or tens of meters.

Deformation bands

Small faults in porous sandstones with displacements of a few mm contain no surface of discontinuity; rather they occur as deformation bands, about 1-mm thick and tens or hundreds of m long across which the displacements are distributed (Fig. 1a). A single deformation band offsetting cross-bedding in the Entrada Sandstone in the San Rafael Desert, appears as a standing rib in the outcrop (Fig. 1a). Examination in thin section (Fig. 1b) shows that sizes of sand grains have been reduced markedly by crushing within the band and that the deformation is highly localized. The average size of grains within the band is about an order of magnitude smaller than that of grains in the parent rock outside the band (Aydin 1978). The average porosity within a band is less than about 6–10%. Since parent sandstones have porosities of about 23–25%, the reduction of porosity during the formation of a deformation band is more than 60%.

Certain observations (Aydin 1978) need to be considered in the development of a suitable theory of deformation banding of porous sandstones.

(1) The deformation is highly localized within a narrow band.

(2) Permanent deformation in a band is by fracturing and displacement of grains, by distortion of the matrix and by reduction of pore volume.

(3) There is both volume decrease and shear displacement across the band. The magnitude of the volumetric strain is at least 0.2, and the average shear strain is of the order of 1–10.

(4) The physical properties, including density and grain size, and probably elasticity and strength, change as deformation proceeds.

Zones of deformation bands

Where two or more deformation bands occur side by side, they form a zone of deformation bands. The zone in the Entrada Sandstone of the San Rafael Desert, Utah, shown in Fig. 1(c), is made up of about ten deformation bands with common dip and strike offsetting a single band of different dip and strike. A zone becomes thicker by addition of new deformation bands; the thickness of a zone depends on the number and spacing of individual deformation bands contained. In the San Rafael Desert, the thickness is as much as about 0.5 m. Total displacement across the zone is the sum of the displacements on individual bands. The average displacement across a zone containing about 100 bands is of the order of 30 cm. The displacement across the zone of about ten bands in the Entrada Sandstone (Fig. 1c) is relatively small, about 2 cm. The individual deformation bands making up a zone are parallel or subparallel, commonly inosculating but rarely crossing (Aydin & Johnson 1978). The traces of individual bands within a zone are quite straight in a plane containing the direction of shear but wavy in a plane normal to the direction of shear.

Certain observations need to be explained by a suitable theory of faulting concerning the production of zones of deformation bands in porous sandstones.

(5) The deformation bands form side by side. This observation raises the question why further deformation is accommodated by the formation of a new band, rather than by continued displacement on a pre-existing band, and why the new band forms immediately adjacent to the pre-existing band.

(6) Zones are at most a few dm wide but are tens or hundreds of meters in extent. Thus zones of deformation bands, like individual deformation bands, represent highly localized deformation of the sandstones.

(7) The trace of an isolated deformation band tends to be straight, but the trace of a deformation band within a zone is wavy or inosculates in a plane normal to the direction of shear.

Slip surfaces

Some of the zones of highly concentrated deformation bands are accompanied by slip surfaces; through-going, discrete surfaces of discontinuity in displacement (Aydin & Johnson 1978). Slip surfaces can be recognized by well-developed striations and grooves (Fig. 1d) that indicate the occurrence and direction of sliding between two blocks. The displacement is much larger across surfaces than across bands and zones (Fig. 1); for example, the net slip along the surface of the Entrada Sandstone shown in Fig. 1(d) is about 7 m. Study of slip surfaces and their adjacent rock by optical and electron microscope indicates that the porosity is nearly zero and the grain size highly reduced within about 0.05 mm of the slip surface (Aydin & Johnson 1978).

Certain observations concerning slip surfaces need to be explained by a suitable theory of faulting of porous sandstones.

(8) A slip surface represents extremely localized, large deformation. The formation of a slip surface marks a change in the style of deformation from continuous and zonal (in a band and in a zone) to discontinuous and planar (in a slip surface).

(9) A slip surface has not been observed in the porous Entrada and Navajo sandstones except within a zone of deformation bands.

Sequential development

Spatial relations among deformation bands, zones of deformation bands, and slip surfaces in the San Rafael Desert, southeastern Utah (Aydin 1978, Aydin & Johnson 1978) indicate that these three structures develop sequentially (Fig. 2). Although slip surfaces are

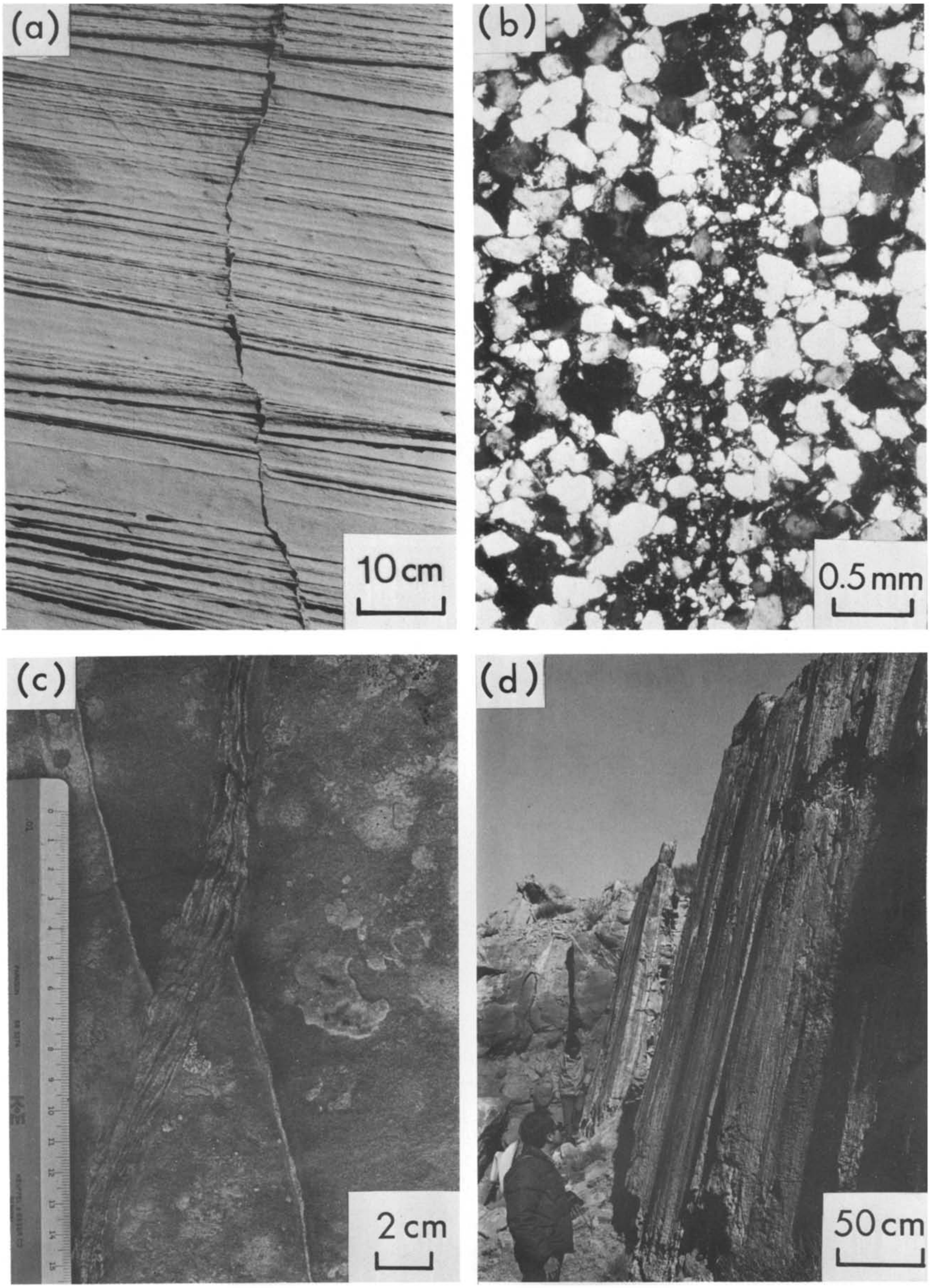

Fig. 1. Three forms of fault in the Entrada Sandstone exposed in the San Rafael Desert, SE Utah. (a) Single deformation band offsetting cross bedding. (b) Photomicrograph showing crushed grains and grain-size reduction within a deformation band. (c) Zone of about 10 deformation bands offsetting a single band. (d) Slip surface with about 7 m of offset. For specific localities and additional examples, see Aydin (1978) and Aydin & Johnson (1978).

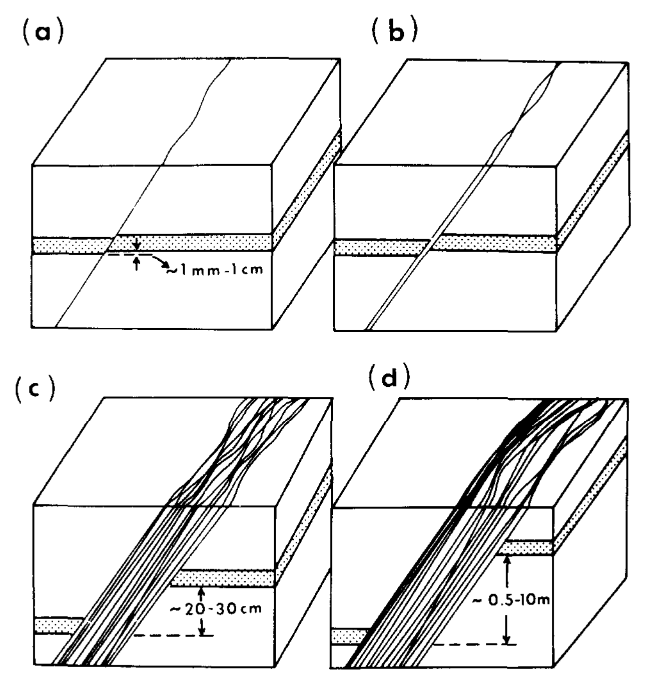

Fig. 2. Series of block diagrams showing sequential development from a single band to a slip surface. (a) Single deformation band. (b) Two inosculating bands. (c) Zone of deformation bands. (d) Slip surface developed on left-hand edge of zone.

always associated with zones of deformation bands, many zones do not contain slip surfaces. Similarly, zones are made up of deformation bands, but deformation bands commonly occur individually, distant from zones. An individual deformation band, therefore, is a primary structural element and its formation marks an early stage in the process of major faulting in the Entrada and Navajo sandstones (Aydin & Johnson 1978).

Multiple bands, zones and surfaces

Deformation bands, zones of deformation bands and slip surfaces occur in networks comprised of multiple sets. Members of a set share a common dip and strike. The spacing between bands within a set is generally 10–25 cm but is larger between zones or surfaces. Networks of multiple sets of bands, zones and surfaces form relatively regular geometric patterns. Such patterns in the San Rafael Desert are described in another paper (Aydin & Reches 1982). Briefly, two sets are common in strike–slip faulting and the strikes of the faults typically intersect at angles of about 60°. There are generally more than two sets, typically four, in dip– and oblique–slip faulting, and the angle between the strikes of the faults ranges from a few tens of degrees to 90° but is typically 20–30°.

These observations of multiple faulting and of the sequence of faulting in porous sandstones need to be explained.

(10) Bands, zones and surfaces of a single set form sequentially and are parallel to one another. The orientation of slip surfaces is controlled by the orientation of zones of deformation bands.

(11) Each type of fault occurs in sets with relatively regular spacing.

(12) Each type of fault occurs in networks constituted of multiple sets. In dip– and oblique–slip faulting, there are more than two and typically four sets.

ANALYSIS OF FAULTING IN POROUS SANDSTONES

The characteristics of deformation bands, zones of deformation bands and slip surfaces observed in the Entrada and Navajo sandstones suggest that faulting in these rocks is not a single process but rather a complex of processes. We are unable to analyze some of the processes, in part because existing theories do not fully accommodate them and in part because experimental studies of such faults do not provide critical parameters. The most promising theories have been developed by Rice, Rudnicki and Cleary (Rice 1975, 1976, Rudnicki & Rice 1975, Cleary & Rudnicki 1976, Cleary 1976, Rudnicki 1977, Rice 1979) who have generalized plasticity theory such that it can describe localized deformation in idealized brittle or plastic materials. We shall use parts of these theories in our analyses of faulting in the Entrada and Navajo sandstones.

We shall analyze in more detail the formation of individual deformation bands. We first describe mechanisms of formation of deformation bands as deduced from field and laboratory observations, then use available theory to quantify some of the mechanisms. We are unable to analyze the formation of zones of deformation bands, but the theory can help us understand the formation of slip surfaces within zones of deformation bands.

Mechanism for formation of deformation bands

Detailed study of terminal parts and outer zones of deformation bands suggests that, in the first stage of deformation, grains move closer together by yielding of matrix and collapsing of pores between grains (Aydin 1978). The band presumably initiates at an imperfection, for example, a small volume of sandstone that contains more pore space than the surrounding sandstone. During the first stage of growth, contact points and contact areas between neighboring grains increase, thereby increasing friction of the mass. This, in turn, results in strong interlocking such that further deformation requires fracturing of grains. The fracturing starts at contacts where high stress concentrations occur (Aydin 1978). The fracturing can result from an increase in pressure alone if the pressure is sufficiently high (Vesic & Clough 1968). It generally results, however, from a combination of pressure and shear, the shear tending to dilate the mass (Rowe 1962), concentrating forces at fewer contact points between grains. After grains fracture, the newly created angular subgrains are further fractured, and most of the original grains are demolished. During this stage, the band decreases in volume and is sheared. Both the strength and the elasticity moduli of the material within the band increase because the decrease in grain size results in an increase in contact points per unit of area and, therefore, a decrease in stress concentration at contact points (e.g. Lambe & Whitman 1969). A band grows in length by crushing of sandstone at its periphery, forming a crudely disc-shaped deformation band about 1-mm thick and several tens or a few hundreds of m in diameter.

A constitutive model for porous sandstones

Most of the essential macroscopic processes involved in the formation of deformation bands in porous sandstones, we believe, are incorporated in a theory of inelastic deformation of strain-hardening (or softening) materials developed by Rudnicki & Rice (1975). The theory is a generalization of the Prandtl–Reuss theory (Hill 1950, pp. 23–45; Kachanov 1974, pp. 58–67) of elastic–plastic, strain-hardening materials. According to the Prandtl–Reuss theory of isotropic hardening, the increase in stress required for an increment of inelastic deformation is some function of the total inelastic strain of the material (Hill 1950, p. 32). If J_2 is the second invariant of the deviatoric stresses,

$$J_2 = (1/2)\sigma'_{ij}\sigma'_{ij} \tag{1a}$$

and

$$\sigma'_{ij} = \sigma_{ij} - (1/3)\sigma_{kk}\delta_{ij}, \tag{1b}$$

where $(1/3)\sigma_{kk}$ is the mean normal stress and δ_{ij} is the Kronecker delta. If Δ^P is the second invariant of the deviatoric strain velocities of inelastic deformation,

$$\Delta^P = (1/2)(D'^P_{ij}D'^P_{ij}) \tag{1c}$$

$$D'_{ij} = D_{ij} - (1/3)D_{kk}\delta_{ij} \tag{1d}$$

$$D_{ij} = (1/2)(\partial v_i/\partial x_j + \partial v_j/\partial x_i) \tag{1e}$$

then

$$J_2 = H[\int \Delta^P \mathrm{d}t], \tag{1f}$$

where H is some function (Fig. 3a), v is a component of velocity, t is time, x is position and the superscript P, refers to plastic or inelastic deformation. The Prandtl–Reuss theory incorporates the Levy–Mises relations, which state that inelastic deformation is incrementally related to deviatoric stresses (Hill 1950, p. 38). An increment of deformation has an elastic component and an inelastic component, and increments of elastic strain and stress are related by the equations (Figs. 3b & c)

$$\mathrm{d}\gamma^e = (\mathrm{d}\tau/G) \tag{2a}$$

$$\mathrm{d}\epsilon^e = -(\mathrm{d}\sigma/K), \tag{2b}$$

where γ is shear strain, τ shear stress, ϵ volumetric strain, σ mean stress, G shear modulus and K bulk modulus. Equations (2) account for elastic deformation of porous sandstone that occurs during loading or unloading of the rock.

The increments of plastic deformation during loading are determined by

$$\mathrm{d}\gamma^P = (\mathrm{d}\tau - \mu\mathrm{d}\sigma)/h; \quad \mathrm{d}\tau - \mu\mathrm{d}\sigma > 0 \tag{3a}$$

$$\mathrm{d}\epsilon^P = -(\mathrm{d}\sigma/K^\circ) + \beta(\mathrm{d}\tau - \mu\mathrm{d}\sigma)/h; \quad \mathrm{d}\sigma > 0. \tag{3b}$$

Here h is a hardening (or softening) modulus, μ coefficient of friction and K° an inelastic bulk modulus that relates inelastic volume change to the change in the mean stress. The bulk modulus is introduced into the Rudnicki–Rice theory in order to accommodate some of our observations. It does not fundamentally change the theory. Equations (3) appear to account for the types of inelastic deformations during loading associated with the growth of a deformation band. The reduction of pore space during the initial stages of formation of a band, for example, can be incorporated through the term K°, which controls the magnitude of the inelastic volume change produced by the mean stress. We imagine that friction results from interlocking as well as contact friction (Rowe 1962) between sand grains. The theory incorporates such effects through the parameter μ.

The method of determining the hardening modulus of a material is explained by Hill (1950, p. 28). If a sample is subjected to pure shear, the hardening modulus relates to the slope, $h^{\tan}$, of the stress–strain curve by

$$h^{\tan} = h/(1 + h/G). \tag{4a}$$

The hardening modulus is a function of the total plastic strain (Fig. 3b) of the material and may be either positive or negative. It is positive if the material is strain hardening and negative if it is strain softening. Finally, β is the

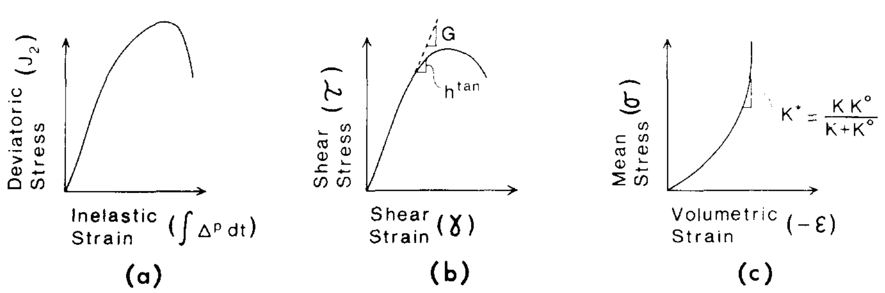

Fig. 3. Idealized stress–strain curves for elastic–inelastic material. (a) Relation between second invariant, J_2, of deviatoric stresses and total deviatoric inelastic strains. (b) Shear stress–strain relation in which slope, $h^{\tan}$, is a function of the elasticity shear modulus, G, and the hardening modulus, h. (c) Relation between mean stress and volumetric strain. K^* is the effective bulk modulus.

dilatancy factor, which relates the increment of inelastic volume change to an increment of inelastic shear strain. The total increment of inelastic volume change resulting from shear and from mean stress can be expressed as

$$d\epsilon^P = -d\sigma/K^\circ + \beta d\gamma^P.$$

Again, the dilatancy factor may be positive or negative. For most dense rocks the dilatancy factor represents opening of cracks and sliding at crack asperities and thus is positive (Rudnicki 1977) whereas for porous rocks, it corresponds to a decrease in volume due to shear and is therefore likely to be negative. Indeed, it is well known in soil mechanics that the sign of dilatancy is determined by the porosity of granular solids and the mean stress acting on them (Lambe & Whitman 1969, p. 131).

All these parameters, the dilatancy factor, the hardening modulus, the inelastic bulk modulus, the friction coefficient, and the two elasticity moduli, will depend on the state of strain, the porosity and the crushing of sand grains so that they are variables, not constants. For example, Morgenstern & Phukan (1970) have shown experimentally that the tangent modulus of a sandstone depends on the porosity of the sandstone.

Equations (2) and (3) can be generalized by replacing shear stress by the second invariant of the deviatoric stresses, strain increment by deformation rates, and stress increments by the spin invariant, Jaumann stress rates (Rudnicki & Rice 1975, p. 378):

$$2D'_{ij} = (\overset{\nabla}{\sigma}'_{ij}/G) + (1/h)(\sigma'_{ij}/J_2)[(\sigma'_{kl}/2)\overset{\nabla}{\sigma}_{kl} + \mu(\overset{\nabla}{\sigma}_{kk}/3)\sqrt{J_2}] \tag{5a}$$

$$D_{kk} = (\overset{\nabla}{\sigma}_{kk}/3K^*) + (\beta/h)[(\overset{\nabla}{\sigma}'_{kl}/2\sqrt{J_2})\overset{\nabla}{\sigma}_{kl} + \mu(\overset{\nabla}{\sigma}_{kk}/3)]. \tag{5b}$$

Here

$$K^* = KK^\circ/(K + K^\circ) \tag{5c}$$

where K^* is the effective bulk modulus.

The Jaumann stress rate (Malvern 1969, Fung 1965) is expressed by

$$\overset{\nabla}{\sigma}_{ij} = (D\sigma_{ij}/Dt) - \sigma_{ip}\Omega_{pj} - \sigma_{jp}\Omega_{pi} \tag{6a}$$

the material time derivative by

$$D\sigma_{ij}/Dt = \partial\sigma_{ij}/\partial t + v_k \partial\sigma_{ij}/\partial x_k \tag{6b}$$

and the spin tensor by

$$\Omega_{ij} = (1/2)[(\partial v_j/\partial x_i) - (\partial v_i/\partial x_j)]. \tag{6c}$$

Equations (5) can be inverted to the form (Rudnicki & Rice 1975, p. 383)

$$\overset{\nabla}{\sigma}_{kl} = \{G[\delta_{mk}\delta_{nl} + \delta_{ml}\delta_{kn}] + [K^* - 2G/3]\delta_{kl}\delta_{mn} - [(G\sigma'_{kl}/\sqrt{J_2}) + \beta K^*\delta_{kl}][(G\sigma'_{mn}/\sqrt{J_2}) + K^*\mu\delta_{mn}]/[h + G + \mu K^*\beta]\}D_{mn} \tag{7a}$$

which is of the form

$$\overset{\nabla}{\sigma}_{kl} = L_{klmn}D_{mn}. \tag{7b}$$

Conditions required for localization of deformation

We shall now investigate conditions for localization of deformation within the idealized material, following Rudnicki & Rice (1975), in order to obtain some insights into possible conditions required for inception of deformation banding in porous sandstones. In Fig. 4, an element of the idealized material made up of matrix surrounding a band infinitely long and of unspecified thickness is subjected to principal stresses σ_1, σ_2 and σ_3 (compressive stresses positive). The coordinate system used for stresses and velocities relates to the orientation of the band. Here x_2 is normal to the band and x_1 and x_3 are in the plane of the band. The stresses inside the band ($\sigma_{ij}^{(i)}$) are shown on the left-hand side in Fig. 4 and the stresses in the matrix ($\sigma_{ij}^{(m)}$) on the right-hand side. At the inception of banding, the velocities of particles remain continuous, and strain rates in the 1- and 3-directions, parallel to the band, remain continuous; any

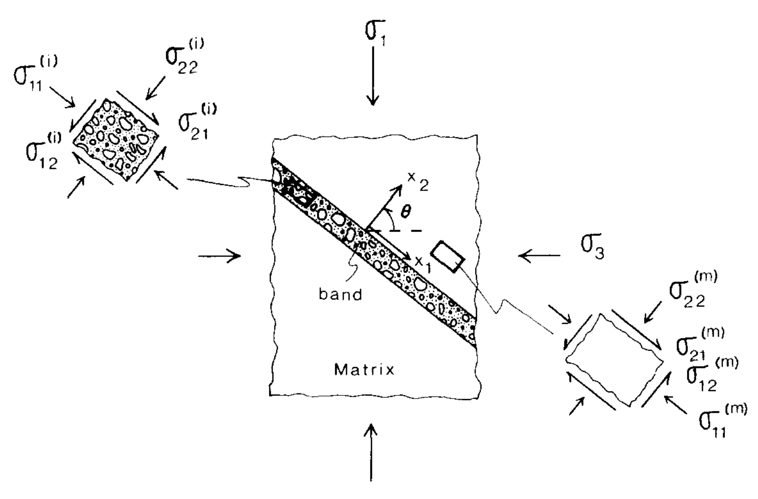

Fig. 4. Idealized band at localization, and identification of stresses and coordinate systems. Superscript *m* refers to matrix and *i* refers to interior of band.

differences between strain rates inside and outside the band are restricted to derivatives of velocities with respect to distances normal to the band (the 2-direction Fig. 4). If $v^{(i)}$ is a velocity inside the band and $v^{(m)}$ a velocity in the matrix, the differences in derivatives of velocities are of the form

$$(\partial v_k^{(i)}/\partial x_l - \partial v_k^{(m)}/\partial x_l) = g_k(x_2)\delta_{l2}, \tag{8a}$$

where g is some function of the 2-direction, which is zero for x_2 outside the band. In investigating conditions for banding, we determine conditions under which there may be differences in strain rates inside a band and in the matrix; that is, we determine conditions under which $g_k(x_2)$ can be nonzero.

In order for there to be equilibrium of stresses at the inception of banding, the material time-rate derivatives of stresses must satisfy the equations

$$\partial(D\sigma_{ij}/Dt)/\partial x_i = 0. \tag{8b}$$

In order for (8b) to be satisfied inside the band and in the matrix and for the differences in strain to be restricted to those defined by (8a), the condition required for equilibrium can be stated as

$$(D\sigma_{2l}^{(i)}/Dt - D\sigma_{2l}^{(m)}/Dt) = 0. \tag{8c}$$

Therefore, the differences in Jaumann stress rates (6a) inside the band and in the matrix are

$$(\overset{\nabla}{\sigma}{}_{21}^{(i)} - \overset{\nabla}{\sigma}{}_{21}^{(m)}) = -(1/2)(\sigma_{22} - \sigma_{11})g_1 + (1/2)\sigma_{13}g_3 \tag{9a}$$

$$(\overset{\nabla}{\sigma}{}_{22}^{(i)} - \overset{\nabla}{\sigma}{}_{22}^{(m)}) = \sigma_{21}g_1 + \sigma_{23}g_3 \tag{9b}$$

$$(\overset{\nabla}{\sigma}{}_{23}^{(i)} - \overset{\nabla}{\sigma}{}_{23}^{(m)}) = -(1/2)(\sigma_{22} - \sigma_{33})g_3 + (1/2)\sigma_{31}g_1. \tag{9c}$$

Now, at the inception of banding, the properties of the material inside the band are the same as those of the material in the matrix so that the coefficients L, in equation (7b), are the same inside and outside the band. Accordingly, the expressions for the differences in Jaumann stress rates inside and outside the band are

$$(\overset{\nabla}{\sigma}{}_{ij}^{(i)} - \overset{\nabla}{\sigma}{}_{ij}^{(m)}) = L_{ijkl}(D_{kl}^{i} - D_{kl}^{m}) = L_{ijk2}g_k. \tag{10a}$$

If we write the right-hand sides of equations (9) as $R_{jk}g_k$, the condition for banding to begin is that

$$\det|L_{2jk2} - R_{jk}| = 0. \tag{10b}$$

The most significant result of this analysis by Rudnicki & Rice (1975) is that localization of deformation can occur; that is, (10b) can be satisfied for certain stress states and for certain ranges of constitutive parameters. Suppose that a material is subjected to a state of uniform stress and strain. If the material has certain combinations of elastic and inelastic properties, the strain can become nonuniform and can become concentrated within a band. The nonuniformity of the strain field is introduced by g_k, and the necessary conditions for the nonuniformity to exist is given by (10b). The condition for banding is the *first* condition reached during the loading history of the material for which (10b) is satisfied. In general, in order to calculate this condition, one must know the functional relations among the states of stress and strain and the various parameters. Unfortunately, there has been no experimental study of rock properties that is sufficiently detailed to define the parameters required for an evaluation of localized deformation.

In their analysis of localization of deformation into bands, Rudnicki & Rice (1975) assume that the hardening modulus, h, changes more with changes of stress and strain than do the shear and bulk moduli G and K, the friction coefficient, μ, and dilatancy factor β. They further assume that the stress–strain curve inherent to the rock itself, without localization phenomena, is concave downward, as shown in Fig. 3(a) so that the hardening modulus is a decreasing function of the amount of inelastic strain. Therefore, they determine the *maximum* value of h that satisfies (10b). If the deviatoric stresses, J_2, are small relative to the elasticity shear modulus, G, of the rock, the second term, R_{jk}, in (10b) is negligible relative to the first, and the condition for localization is that

$$\det|L_{2jk2}| = 0$$

and the hardening modulus must satisfy the equation

$$h/(G + \mu K^*\beta) = -1 + A + B, \tag{11a}$$

where

$$A = (G\sigma_{22}' + \beta K^*\sqrt{J_2})(G\sigma_{22}' + \mu K^*\sqrt{J_2})/ [J_2(K^* + 4G/3)(G + \mu K^*\beta)] \tag{11b}$$

$$B = G(\sigma_{2l}^2 + \sigma_{23}^2)/[J_2(G + \mu K^*\beta)]. \tag{11c}$$

Now, (11a) contains the stresses σ_{ij} referred to the coordinate system of the band (Fig. 4), which can be expressed in terms of magnitude and directions of principal stresses relative to the direction of the band. By differentiating the hardening modulus in (11a) with respect to the three direction cosines that relate the orientation of the band and the principal stress directions, Rudnicki & Rice (1975, appendix) derived the following results.

(1) The orientation of band that maximizes the hardening modulus contains the intermediate principal stress; that is, the direction of σ_2 lies within the band. This result is not at all encouraging for an explanation of multiple sets of deformation bands in the Navajo and Entrada sandstones (Observation 12 in earlier paragraphs). The theory would predict two conjugate bands, whereas the field observations indicate more than two, typically four, for dip– and oblique–slip faulting. It is possible that experimental determination of the parameters will provide a means for eliminating this critical difficulty.

(2) The orientation of the band that maximizes h is a function of the stress state as well as the various elasticity and inelasticity parameters (Fig. 4):

$$\theta_{crit} = \theta_0 = \tan^{-1}[(\xi - \sigma_3'/\sqrt{J_2})/\{(\sigma_1'/\sqrt{J_2}) - \xi\}]^{1/2} \quad (12a)$$

where

$$\xi = [(1 + \nu^*)(\beta + \mu)/3] - (\sigma_2'/\sqrt{J_2})(1 - \nu^*) \quad (12b)$$

$$\nu^* = [(3K^*/2G) - 1]/[(3K^*/G) + 1]. \quad (12c)$$

Rudnicki & Rice (1975) have shown that (12a) predicts bands oriented at angles less than 45° to the direction of maximum compression (as in the Coulomb and Mohr theories of faulting) if the dilatancy factor is positive.

(3) The corresponding critical hardening modulus is

$$(h_{crit}/G) = h_0/G = (1 + \nu^*)[(\beta - \mu)^2/\{9(1 - \nu^*)\} - (1/2)\{(\sigma_2'/\sqrt{J_2}) + (\beta + \mu)/3\}^2]. \quad (12d)$$

If the deviatoric stresses are comparable in magnitude to the shear modulus, the expressions for the critical hardening modulus and the orientation of the bands are more complicated. To first order in J_2/G (Rudnicki & Rice 1975, p. 390),

$$\theta_{crit} = \theta_0 + (\sqrt{J_2}/G)[(\mu - \beta)(1 + \nu^*)\,\mathrm{ctn}(2\theta_0)/6 - (1/4)\{1 - (3/4)\sigma_2'^2/J_2\}^{1/2}\sin(2\theta_0)] \quad (12e)$$

and

$$(h_{crit}/G) = h_0/G + (4 - 3\sigma_2'^2/J_2)(\mu - \beta)\sin^2(2\theta_0)(\sqrt{J_2}/G)/\{24(1 - \nu^*)\}, \quad (12f)$$

where θ_0 is defined in (12a) and h_0/G in (12d).

Rudnicki & Rice (1975) have shown that, depending on the nature of the far-field stress state and of the constitutive parameters, localization in the form of deformation bands may set in either while the rock mass is continuously hardening, in which case h_{crit} is positive, or when it is past its peak strength and softening, in which case h_{crit} is negative. Their results show that, in general, if the dilatancy factor (β) is positive, rocks subjected to 'triaxial' or 'uniaxial' compression should not develop localization in the form of bands unless the material is well beyond its peak strength and markedly softening. For example, the lowermost curve in Fig. 5, based on (12d), is for an axisymmetric stress state in which $\sigma_2 = \sigma_3$, so that $\sigma_2'/\sqrt{J_2} = 1/\sqrt{3}$. The critical tangent hardening modulus,

$$h_{crit}^{tan} = h_{crit}/(1 + h_{crit}/G), \quad (13)$$

ranges from about $-0.5G$ to $-1.18G$ for a dilatancy factor, β, ranging from 0 to 0.5, so that the material must be softening markedly in order for localization to occur. If the deviatoric intermediate stress is zero, however, as in the case of pure shear, the tangent hardening modulus ranges from about $+0.03G$ to $-0.08G$ for the same range of dilatancy factor.

Examination of deformation bands in the Entrada and Navajo sandstones suggests that there is volume change in response to changes in mean stress and that dilatancy factors can be negative; that is, that shear can cause a volume decrease in the sandstones. First let us examine consequences of negative values of the dilatancy factor.

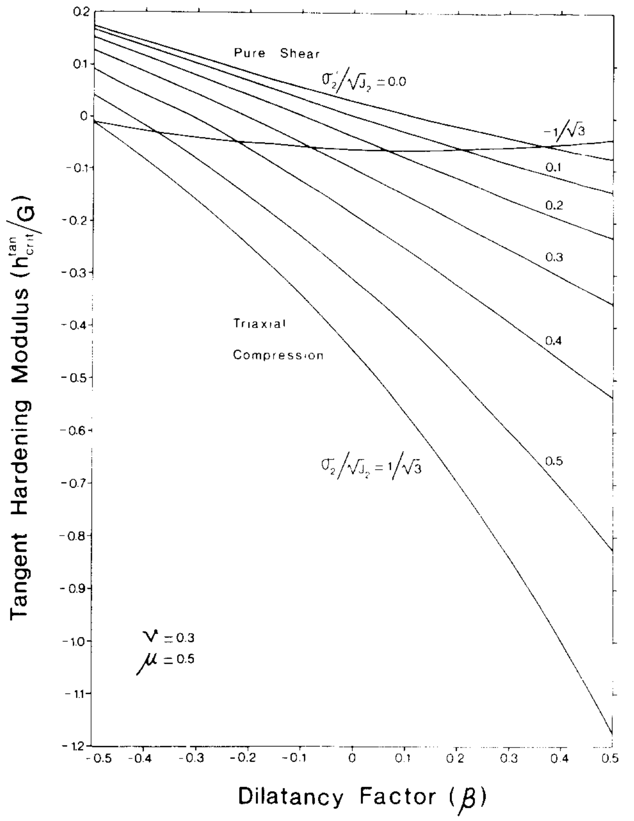

Fig. 5. Critical hardening modulus, h_{crit}^{tan}, normalized with shear modulus, G, as a function of dilatancy factor, β, for various stress states. Positive values of hardening modulus correspond to hardening materials and negative to softening materials.

Figure 5 shows that negative dilatancy factors can markedly increase the critical, tangent hardening moduli. For axisymmetric stress states, for example, the critical hardening modulus approaches zero as the dilatancy factor approaches -0.5. For zero values of intermediate deviatoric stress, as for example in pure shear, the critical hardening moduli are positive for all negative values of the dilatancy factor.

The effect of the inelastic volume change that results from changes of the mean stress can be studied by varying the parameter K^* in equation (12c). K^* is defined in (5c) in terms of the stiffness, $K°$, relative to inelastic volume changes produced by changes in mean stress and of the elasticity bulk modulus, K. It is clear that

$$K^* \to K \quad \text{if} \quad K° \gg K$$

and

$$K^* \to K° \quad \text{if} \quad K° \ll K.$$

Further, if Poisson's ratio is 0.3, and $K°$ is so large that the material is essentially incompressible inelastically, then $K^*/G = 2.17$. The upper and lower curves in Figs. 6(a) & 8(b), respectively, closely correspond with this case. As the material becomes more compressible inelastically, K^*/G reduces, approaching zero. The corresponding magnitudes of the critical hardening modulus decrease as K^*/G decreases. These results indicate that localization tends to occur for stresses near peak values, where h_{crit}^{tan} is zero.

The crushing of grains within deformation bands in the Navajo and Entrada sandstones produces volumetric strains of at least 0.2 (volume decrease), and the average shear strain within a band is at least unity. Based on these data, we presume that the dilatancy factor is negative, perhaps −0.2 or smaller. For a dilatancy factor of −0.2, according to Fig. 6, the critical hardening moduli should range from about −0.25 to 0.1. Localization should occur near the peak of the stress–strain curves for these sandstones. It is likely that the dilatancy factor is smaller than −0.2, because the volume change might be much greater than the value used to calculate this factor (Aydin 1978), so that the critical hardening modulus for the onset of deformation banding in the Entrada and Navajo sandstones probably is positive.

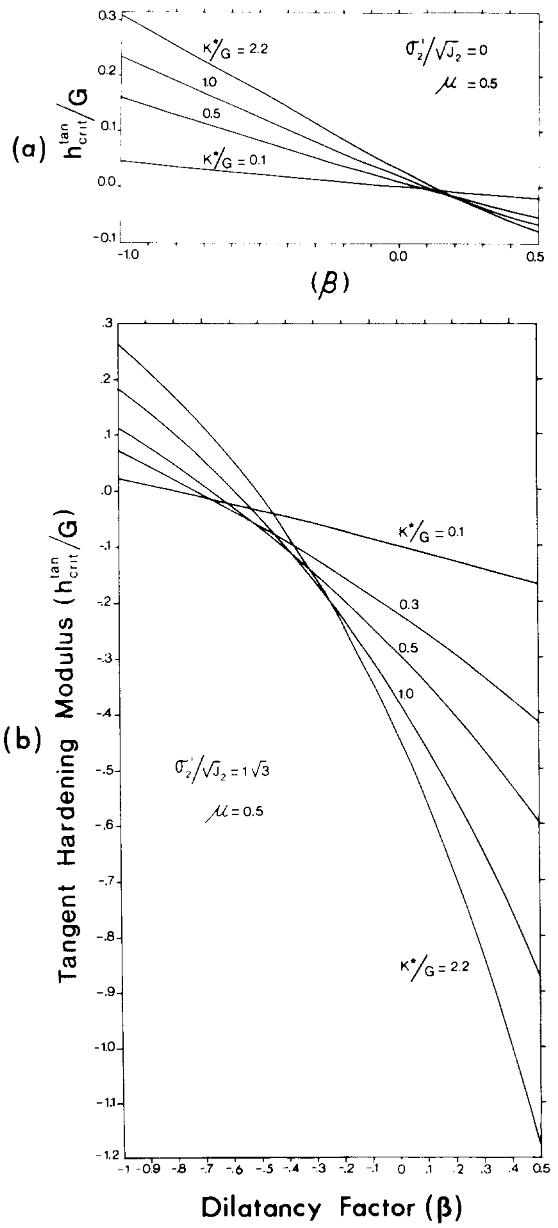

Fig. 6. Critical hardening modulus, h^{tan}_{crit}, normalized with shear modulus, G, as a function of dilatancy factor, β, for various effective bulk moduli, K^*. (a) Zero deviatoric intermediate principal stress, for example pure shear. (b) Axisymmetric stress states, such as 'uniaxial' and 'triaxial' experimental testing.

Development of zones of deformation bands

The formation of deformation bands side by side in the form of zones probably results partly from strain hardening of the crushed material within a deformation band. The pore volume is much lower in a deformation band in the Entrada and Navajo sandstones than in the parent rock, and the decrease in pure volume should affect at least two of the constitutive parameters discussed here, the dilatancy factor, β, and the inelastic bulk modulus, K°. The dilatancy factor (3b) for porous sandstones is probably negative during initiation of a band, as indicated before, but becomes positive for material within a band as pores collapse and grains interlock there. The inelastic bulk modulus (3b), which relates inelastic volume change to change in the mean stress, certainly increases markedly as the pore volume decreases within the band. The effective bulk modulus, K^* (5c), of the crushed sandstone within bands, therefore, probably exceeds the elastic bulk modulus of the parent sandstone. It is also possible that the hardening modulus h increases within the band. There are indeed several reasons to believe that the material within the band strain hardens relative to the sandstone on either side of the band and that other bands must be initiated to accommodate further faulting (Aydin 1978, pp. 926–927).

It is not clear why the bands tend to occur close together and to inosculate to form a zone. Perhaps the formation of a zone is partly a result of interaction among nearby bands.

Development of slip surfaces

Slip surfaces in the Entrada and Navajo sandstones always occur in parts of well-developed zones of deformation bands where these bands are highly concentrated. Since development of faults as slip surfaces in these rocks appears to mark a change in style of faulting, it seems appropriate to consider slip surfaces as manifestations of some kind of mechanical instability. We believe that the formation of slip surfaces within zones can be understood in terms of the conditions of 'runaway instability' (Rudnicki 1977, Rice 1979) which can be understood qualitatively in terms of the 'Eshelby line' that relates stresses in the matrix to stresses in an inclusion.

According to Rudnicki (1977), runaway instability is a condition where strain increments within an inclusion become boundlessly large with an increase in the far-field stresses. Whereas the analysis of deformation bands was based on bifurcation in the solution for velocities within a band, the analysis of runaway instability is based on boundless velocities within an inclusion. The stresses within a homogeneous inclusion are homogeneous, as shown by Eshelby (1957, 1959), even if the inclusion is anisotropic, so that the entire inclusion becomes unstable during runaway instability. Rudnicki (1977) pointed out that this result pertains even if the inclusion deforms nonlinearly; he has determined the hardening modulus required for runaway instability within a disc-shaped inclusion composed of the elastic–inelastic material discussed in this paper, see equations (2)–(5). The expression for the hardening modulus for instability is similar to that for bifurcation of solutions for an infinitely long inclusion (equation 11) except for some correction factors. Thus

$$h_r/(G^{(i)} + \mu K^*\beta) = -1 + (1-\eta)A + [1 - \zeta g^{(i)}/(K^* + 4G^{(i)}/3)]B;\ (b/a) \ll 1, \quad (14a)$$

where A and B are defined in (11b and 11c), except that the σ's in (11) are replaced with corresponding values for the inclusion:

$$\sigma'_{22} \rightarrow \sigma'^{(i)}_{22};\ \sigma'_{11} \rightarrow \sigma'^{(i)}_{21};\ \sigma'_{23} \rightarrow \sigma'^{(i)}_{23}$$

and

$$\eta = (b/a)\pi(2-\nu)/(1-\nu) \quad (14b)$$

$$\zeta = (b/a)\pi/(1-\nu). \quad (14c)$$

Examination of (14a) for a finite, disc-shaped inclusion indicates that two factors, in addition to those discussed in connection with bifurcation and localization of deformation, affect the value of hardening modulus required for runaway instability. The first factor is the length of the inclusion. The longer the inclusion, the more unstable it is. This reduction is apparent in the correction factors, (14b) and (14c), which contribute to negative terms in the expression for the hardening modulus, (14a). The second factor is the relative magnitude of stresses in the inclusion and matrix. The stresses that enter (14a) are the stresses within the inclusion, which can be less than or greater than the far-field stresses, depending on whether the inclusion is softer or stiffer than the matrix. Presumably, runaway instability will tend to occur in stiffer inclusions, such as zones of deformation bands, rather than in relatively soft matrix, and the longer the zones, the more unstable they are.

Runaway-instability within stiff inclusions such as zones of deformation bands in sandstone can be qualitively understood by means of a procedure suggested by Rudnicki (1977) and Rice (1979) based on equations derived by Eshelby. According to Eshelby (1957, 1959), the difference between the shear stress in the inclusion and the shear stress in the far-field, for example, is proportional to the difference between the shear strain in the inclusion and strain in the far-field,

$$\sigma^{(i)}_{12} - \sigma^{\infty}_{12} = 2G(1 - 1/S_{1212})(\epsilon^{(i)}_{12} - \epsilon^{\infty}_{12}). \quad (15)$$

The shape factor, S_{1212}, assumes the following values for Poisson's ratio of 0.2:

a/b	S_{1212}
1.1	0.247
5	0.387
10	0.426
100	0.492
1000	0.499

Here a/b is the diameter-to-thickness ratio of the disc-shaped inclusion. Equation (15) is valid regardless of the properties of the inclusion, as pointed out by Rudnicki (1977) and by Rice (1979), but the matrix must be linearly elastic. The properties of the inclusion, however, determine the magnitude of the strain, $\epsilon^{(i)}_{12}$, in the inclusion.

The significance of (15) can best be illustrated with an example. The stress–strain curve for the matrix is essentially linear (O'–C', Fig. 7). The crushed sandstone within the zone has been highly loaded and deformed inelastically as well as elastically, and then partly unloaded, as discussed in connection with the growth of deformation bands. The loading curve is assumed to be of the form O–B–C (Fig. 7) and the unloading curve is O'–A–B. The origin of coordinates for strain within the zone is selected to be where the unloading curve intersects the strain axis (Fig. 7) so that the strains we will consider within the zone are elastic until point B is reached. Since we have deduced that the zone is stiffer than the matrix (Aydin & Johnson 1978), the slope of line O'–B is steeper than that of line O'–B'. Now suppose that the far-field stresses are represented by A' (Fig. 7). Then the stress within the zone is higher, represented by A, according to (15), because the magnitude of the shear strain within the inclusion is less than that of the far-field, and the coefficient $1 - 1/S_{1212}$ is always negative, as indicated by the tabulated values of S_{1212}. As the far-field stresses are increased to a value

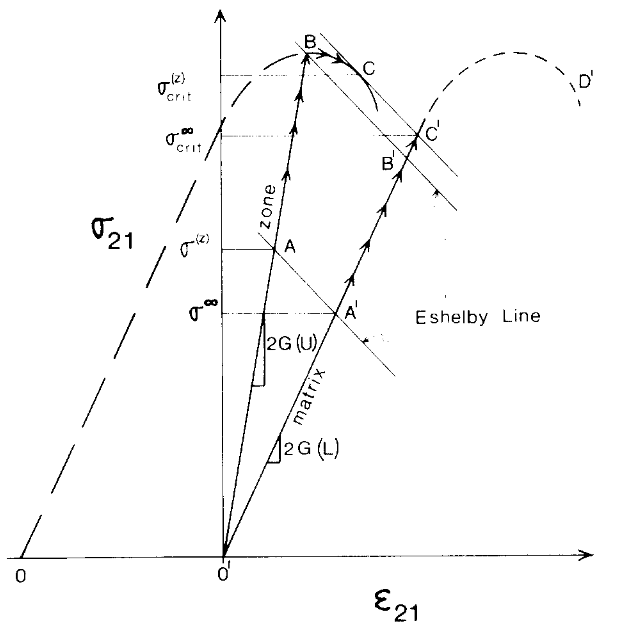

Fig. 7. Possible conditions during formation of a slip surface within a zone of deformation bands in the Entrada or Navajo sandstones. Curve O–B–C represents stress–strain relation for material within zone, curve O'–A'–B'–C'–D for material in matrix. During formation of the zone, the stress and strain reached relatively high values at point B; stress then relaxed to A. The zone is stiffer than the matrix [$G(U) >$ $G(L)$] so that the far-field stress, σ^{∞}, is lower, A', than the stress, $\sigma^{(z)}$, in the zone, as indicated in equation (15), which defines the 'Eshelby line.' As the far-field stress increases to C', the stress in the zone passes through a maximum and approaches the critical value, $\sigma^{(z)}_{\text{crit}}$, at which point the deformation in the zone becomes *unstable* because the far-field stress must decrease with further increase of strain within the zone.

corresponding to C' (Fig. 7), the shear stress within the zone becomes equal to the critical value, $\sigma_{crit}^{(z)}$, represented by C.

At this point, further deformation cannot be accommodated quasistatically by the material within the zone, and the material is dynamically driven to large strains. We imagine that this condition leads to the formation of a slip surface within the zone. The zone is stable for far-field stresses between the values represented by points B' and C' (Fig. 9), even though the stress–strain curve for the material within the zone is falling, because the far-field stress must increase in order for the strain to increase within the zone.

The instability represented by Rudnicki's 'runaway instability' and by Rice's 'Eshelby line' (15) is an interaction between the inelastic properties of the material and the 'softness' of the loading system, in this case the matrix. In effect, the relatively soft matrix behaves much as a 'soft' loading machine used in testing rocks in which a rock specimen fails violently as strain energy is released from the loading machine (e.g. Wawersik & Fairhurst 1970). The instability recognized by Rudnicki is an attractive explanation, we believe, for the formation of slip surfaces within zones of deformation bands.

CONCLUSIONS

Study of faults in the form of deformation bands, zones of deformation bands and slip surfaces in the Navajo and Entrada sandstones led to several questions concerning the mechanisms and mechanics of faulting in porous rocks. We have addressed some of these questions qualitatively in terms of a theory of deformation of elastic–plastic, time-independent materials developed in a series of papers by Rice, Rudnicki and Cleary (for a compact summary see Rice 1979) and in terms of inclusion theory developed by Eshelby (1957).

The tendency for strain to localize in porous rocks can be understood in terms of bifurcation in the macroscopic constitutive relations. The terms within the constitutive relations account for the principal field and laboratory observations of faults in sandstones: there is inelastic volume change and shear within bands and the physical properties of the material change as deformation proceeds.

The deformation bands in the Entrada and Navajo sandstones occur in multiple sets, typically four for dip– and oblique–slip faulting. We cannot explain the patterns of the multiple sets of bands in terms of the analyses. The analysis of localization by Rudnicki & Rice (1975) indicates that infinitely long shear bands should form in conjugate pairs and that the bands should contain the direction of the intermediate principal stress. All other theories of faulting with which we are familiar have the same character and are further restricted to plane–strain deformations (e.g. Varnes 1962, Odé 1960). The theory by Rudnicki & Rice is not restricted to plane–strain deformations and might predict multiple sets of faults, depending on the variations of the many parameters. Oertel (1965) has demonstrated that multiple faults form essentially simultaneously, at least in some materials, and Reches (1978) has shown that if strain is primarily by slip along faults, four sets are required kinematically to accommodate three-dimensional strains and rotations.

The formation of several parallel or subparallel deformation bands rather than continued strain within a single band can be understood in terms of strain hardening. Our analyses, however, have no bearing on the formation of bands side by side as in well-developed zones of deformation bands.

Slip surfaces in the Entrada and Navajo sandstones always occur within a well-developed zone of deformation bands, and they represent extremely localized, large deformation. The formation of slip surfaces might be understood in terms of 'runaway instability' (Rudnicki 1977, Rice 1979) wherein the strains within a zone become indefinitely large as the far-field stress state becomes critical. Because the material within a zone is stiffer than the surrounding sandstone, the stresses within the zone are higher than the far-field stresses. Therefore, the surrounding sandstone can be deforming elastically under relatively low far-field stresses, whereas the stresses within the zone can be sufficiently high for the strains to become extremely large. Apparently the large strains are accommodated along a discrete surface of discontinuity.

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